**Problem 1**

**Algorithm**

Note: A hospital will be matched iff all its positions are filled

GALE–SHAPLEY (list of hospitals with their preference lists and number of positions, list of students with their preference list)

INITIALIZE M to empty matching.

WHILE (some hospital h is unmatched and has not proposed to every student)

FOR (the number of empty positions that hospital h has)

s ← first student on h’s list to whom h has not yet proposed.

IF (s is unmatched)

Add h–s to matching M.

Update empty positions of h

ELSE IF (s prefers h to current partner hʹ)

Replace hʹ–s with h–s in matching M.

Update empty positions of h and h’s

ELSE

s rejects h.

RETURN stable matching M.

**Proof that all hospitals are matched:**

* For the sake of contradiction, suppose that after termination of Gale-Shapley algorithm a hospital is h still unmatched.
* After termination of algorithm, there are still some unmatched students left since there are more students than the number of positions
* Any student that remains unmatched is because no hospital including h has proposed to it.
* But algorithm makes sure that hospital h proposes to every student if is not matched.
* Hence, a contradiction.

**Proof that all the matches are stable:**

Suppose a pair h-s that didn’t make it to the matching M returned by algorithm

*Case 1:*

* h never proposed to s
* h preferred all of its gale shapely partners to s
* Hence, no chance of side deal between h-s so h-s is stable

*Case 2:*

* h proposed to s
* s rejected h right away or later by accepting another hospital h’ that was higher in his preference list than h
* Hence, s prefers h’ over h
* Again, no chance of side deal between h-s so h-s is stable

In either case, h-s is stable which means that all the pairs in M are stable because no student and hospital can make a side deal and hence, create instability.